$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})=\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})\,, \tag{1}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\mathbf{a}\cdot\mathbf{c}\,\mathbf{b}-\mathbf{a}\cdot\mathbf{b}\,\mathbf{c}\,, \qquad (\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\mathbf{a}\cdot\mathbf{c}\,\mathbf{b}-\mathbf{b}\cdot\mathbf{c}\,\mathbf{a}\,. \tag{2}$$

$$\nabla\times\nabla\Phi=0\,, \tag{3}$$

$$\nabla\cdot(\nabla\times\mathbf{a})=0\,, \tag{4}$$

$$\nabla\times(\nabla\times\mathbf{a})=\nabla\,\nabla\cdot\mathbf{a}-\nabla^2\mathbf{a}\,. \tag{5}$$

$$\nabla\cdot(\Phi\,\mathbf{a})=\mathbf{a}\cdot\nabla\Phi+\Phi\,\nabla\cdot\mathbf{a}\,, \tag{6}$$

$$\nabla\times(\Phi\,\mathbf{a})=\nabla\Phi\times\mathbf{a}+\Phi\,\nabla\times\mathbf{a}\,, \tag{7}$$

$$\mathbf{a}\times(\nabla\times\mathbf{b})=(\nabla\mathbf{b})\cdot\mathbf{a}-\mathbf{a}\cdot\nabla\mathbf{b}\,, \tag{8}$$

$$(\mathbf{a}\times\nabla)\times\mathbf{b}=(\nabla\mathbf{b})\cdot\mathbf{a}-\mathbf{a}\,\nabla\cdot\mathbf{b}\,, \tag{9}$$

$$\nabla(\mathbf{a}\cdot\mathbf{b})=\mathbf{a}\cdot\nabla\mathbf{b}+\mathbf{b}\cdot\nabla\mathbf{a}+\mathbf{a}\times(\nabla\times\mathbf{b})+\mathbf{b}\times(\nabla\times\mathbf{a})\,, \tag{10}$$

$$\nabla\cdot(\mathbf{a}\,\mathbf{b})=\mathbf{a}\cdot\nabla\mathbf{b}+\mathbf{b}\,\nabla\cdot\mathbf{a}\,, \tag{11}$$

$$\nabla\cdot(\mathbf{a}\times\mathbf{b})=\mathbf{b}\cdot\nabla\times\mathbf{a}-\mathbf{a}\cdot\nabla\times\mathbf{b}\,, \tag{12}$$

$$\nabla\times(\mathbf{a}\times\mathbf{b})=\mathbf{a}\,\nabla\cdot\mathbf{b}+\mathbf{b}\cdot\nabla\mathbf{a}-\mathbf{b}\,\nabla\cdot\mathbf{a}-\mathbf{a}\cdot\nabla\mathbf{b}\,. \tag{13}$$

$$\iiint \nabla \cdot \mathbf{a}\, d\tau = \oiint \mathbf{a} \cdot \mathbf{n}\, d\sigma \qquad \text{(Gauss)}\,, \tag{14}$$

$$\mathbf{a} \to \mathbf{a} \times \mathbf{c}(\text{onst}) \quad \Rightarrow \quad \iiint \nabla \times \mathbf{a}\, d\tau = \oiint \mathbf{n} \times \mathbf{a}\, d\sigma\,, \tag{15}$$

$$\mathbf{a} \to \Phi\, \mathbf{c}(\text{onst}) \quad \Rightarrow \quad \iiint \nabla\Phi\, d\tau = \oiint \Phi\, \mathbf{n}\, d\sigma\,, \tag{16}$$

$$\mathbf{a} \to \Phi\nabla\Psi - \Psi\nabla\Phi \;\Rightarrow$$

$$\iiint (\Phi\nabla^2\Psi - \Psi\nabla^2\Phi)\, d\tau = \oiint (\Phi\nabla\Psi - \Psi\nabla\Phi) \cdot \mathbf{n}\, d\sigma \quad \text{(Green)}\,, \tag{17}$$

$$\iint (\nabla \times \mathbf{a}) \cdot \mathbf{n}\, d\sigma = \oint \mathbf{a} \cdot d\boldsymbol{l} \qquad \text{(Stokes)}\,, \tag{18}$$

$$\mathbf{a} \to \mathbf{a} \times \mathbf{c}(\text{onst}) \quad \Rightarrow \quad \iint (\mathbf{n} \times \nabla) \times \mathbf{a}\, d\sigma = \oint d\boldsymbol{l} \times \mathbf{a}\,, \tag{19}$$

$$\mathbf{a} \to \Phi\, \mathbf{c}(\text{onst}) \quad \Rightarrow \quad \iint \mathbf{n} \times \nabla\Phi\, d\sigma = \oint \Phi\, d\boldsymbol{l}\,. \tag{20}$$

Maxwell's equations

$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$	Faraday
$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	'Ampère'
$\nabla \cdot \mathbf{B} = 0$	no magnetic monopoles
$\nabla \cdot \mathbf{E} = \frac{\tau}{\epsilon_0}$	Poisson

One cannot escape the feeling that these equations have an existence and intelligence of their own; that they are wiser than we are, wiser even than their discoverers; that we get more out of them than was originally put into them.

(Heinrich Hertz)

Physical constants

Physical quantity	*Symbol*	*Value*	*Units*
charge of the electron	e	1.602×10^{-19}	C
mass of the electron	m_e	9.109×10^{-31}	kg
mass of the proton	m_p	1.673×10^{-27}	kg
permittivity of the vacuum	ϵ_0	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
permeability of the vacuum	μ_0	1.257×10^{-6} $(= 4\pi \times 10^{-7})$	$\mathrm{H\,m^{-1}}$
velocity of light	c	2.998×10^{8} $(= (\epsilon_0\mu_0)^{-1/2})$	$\mathrm{m\,s^{-1}}$
Planck's constant	h	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann's constant	k	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
gravitational constant	G	6.671×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Physical quantity	*Definition*	*Expression**	*Units*
plasma frequency	$\omega_{p,e} \equiv \sqrt{\frac{ne^2}{\epsilon_0 m_e}}$	$= 56.6\sqrt{n}$	$\mathrm{rad\,s^{-1}}$
Debye length	$\lambda_D \equiv \sqrt{\frac{\epsilon_0 kT}{ne^2}}$	$= 69.0\sqrt{\frac{T}{n}}$	m
electron cyclotron frequency	$\Omega_e \equiv \frac{eB}{m_e}$	$= 1.76 \times 10^{11}\, B$	$\mathrm{rad\,s^{-1}}$
electron cyclotron radius	$R_e \equiv \frac{v_{\perp,e}}{\Omega_e}$	$= 2.21 \times 10^{-8}\,\frac{\sqrt{T}}{B}$	m
ion cyclotron frequency	$\Omega_i \equiv \frac{ZeB}{m_i}$	$= 9.58 \times 10^{7}\,\frac{Z}{A}\,B$	$\mathrm{rad\,s^{-1}}$
ion cyclotron radius	$R_i \equiv \frac{v_{\perp,i}}{\Omega_i}$	$= 9.47 \times 10^{-7}\,\frac{\sqrt{A}}{Z}\,\frac{\sqrt{T}}{B}$	m
electron thermal speed	$v_{\mathrm{th},e} \equiv \sqrt{\frac{2kT}{m_e}}$	$= 5.5 \times 10^{3}\sqrt{T}$	$\mathrm{m\,s^{-1}}$
ion thermal speed	$v_{\mathrm{th},i} \equiv \sqrt{\frac{2kT}{m_i}}$	$= 1.3 \times 10^{2}\,\frac{1}{\sqrt{A}}\,\sqrt{T}$	$\mathrm{m\,s^{-1}}$
sound speed	$v_s \equiv \sqrt{\frac{\gamma p}{\rho}}$	$= 1.17 \times 10^{2}\sqrt{\frac{1+Z}{A}}\,\sqrt{T}$	$\mathrm{m\,s^{-1}}$
Alfvén speed	$v_A \equiv \frac{B_0}{\sqrt{\mu_0\rho}}$	$= 2.18 \times 10^{16}\sqrt{\frac{Z}{A}}\,\frac{B}{\sqrt{n}}$	$\mathrm{m\,s^{-1}}$

	Physical quantity	*Symbol/definition*	**Tokamak**	**Coronal loop**	**Solar wind**	**Magnetosphere**
⇒	particle density	n	$10^{20}\,\mathrm{m}^{-3}$	$10^{16}\,\mathrm{m}^{-3}$	$10^{7}\,\mathrm{m}^{-3}$	$10^{10}\,\mathrm{m}^{-3}$
⇒	magnetic field	B	3 T (30 kG)	0.03 T (300 G)	6×10^{-9} T (60 μG)	3×10^{-5} T (0.3 G)
⇒	temperature	T	$10^{8}\,\mathrm{K}\ (\approx 10\,\mathrm{keV})$	$10^{6}\,\mathrm{K}\ (\approx 100\,\mathrm{eV})$	$10^{5}\,\mathrm{K}\ (\approx 10\,\mathrm{eV})$	$10^{4}\,\mathrm{K}\ (\approx 1\,\mathrm{eV})$
	electron th. speed	$v_{\mathrm{th},e} \equiv \sqrt{2kT/m_e}$	$5.9\times10^{7}\,\mathrm{m\,s^{-1}}$	$5900\,\mathrm{km\,s^{-1}}$	$1700\,\mathrm{km\,s^{-1}}$	$590\,\mathrm{km\,s^{-1}}$
	ion thermal speed*	$v_{\mathrm{th},i} \equiv \sqrt{2kT/m_i}$	$1.4\times10^{6}\,\mathrm{m\,s^{-1}}$	$140\,\mathrm{km\,s^{-1}}$	$41\,\mathrm{km\,s^{-1}}$	$14\,\mathrm{km\,s^{-1}}$
	electron gyro freq.	$\Omega_e \equiv eB/m_e$	$5.3\times10^{11}\,\mathrm{rad\,s^{-1}}$	$5.3\times10^{9}\,\mathrm{rad\,s^{-1}}$	$1.1\times10^{3}\,\mathrm{rad\,s^{-1}}$	$5.3\times10^{6}\,\mathrm{rad\,s^{-1}}$
		$\Omega_e/(2\pi)$	$84\,\mathrm{GHz}$	$0.84\,\mathrm{GHz}$	$0.17\,\mathrm{kHz}$	$0.84\,\mathrm{MHz}$
	electron gyro radius	$R_e \equiv v_{\perp,e}/\Omega_e$	$0.1\,\mathrm{mm}$	$1\,\mathrm{mm}$	$1.5\,\mathrm{km}$	$10\,\mathrm{cm}$
	ion gyro freq.*	$\Omega_i \equiv ZeB/m_i$	$2.9\times10^{8}\,\mathrm{rad\,s^{-1}}$	$2.9\times10^{6}\,\mathrm{rad\,s^{-1}}$	$0.58\,\mathrm{rad\,s^{-1}}$	$2.9\times10^{3}\,\mathrm{rad\,s^{-1}}$
		$\Omega_i/(2\pi)$	$46\,\mathrm{MHz}$	$0.46\,\mathrm{MHz}$	$0.1\,\mathrm{Hz}$	$0.46\,\mathrm{kHz}$
	ion gyro radius*	$R_i \equiv v_{\perp,i}/\Omega_i$	$4.9\,\mathrm{mm}$	$4.9\,\mathrm{cm}$	$71\,\mathrm{km}$	$4.9\,\mathrm{m}$
	plasma frequency	$\omega_{p,e} \equiv \sqrt{ne^2/(\epsilon_0 m_e)}$	$5.7\times10^{11}\,\mathrm{rad\,s^{-1}}$	$5.7\times10^{9}\,\mathrm{rad\,s^{-1}}$	$1.8\times10^{5}\,\mathrm{rad\,s^{-1}}$	$5.7\times10^{6}\,\mathrm{rad\,s^{-1}}$
		$\omega_{p,e}/(2\pi)$	$91\,\mathrm{GHz}$	$0.91\,\mathrm{GHz}$	$29\,\mathrm{kHz}$	$0.91\,\mathrm{MHz}$
	electron skin depth	$\delta_e \equiv c/\omega_{p,e}$	$0.53\,\mathrm{mm}$	$5.3\,\mathrm{cm}$	$1.7\,\mathrm{km}$	$53\,\mathrm{m}$
	Debye length	$\lambda_D \equiv \sqrt{\epsilon_0 kT/(ne^2)}$	$0.07\,\mathrm{mm}$	$0.7\,\mathrm{mm}$	$7\,\mathrm{m}$	$7\,\mathrm{cm}$

	Physical quantity	Symbol/definition	Tokamak	Coronal loop	Solar wind	Magnetosphere
⇒	width	a	$1\,\mathrm{m}$	$10\,000\,\mathrm{km}$	$1.5\times10^{6}\,\mathrm{km}$	$6\times10^{3}\,\mathrm{km}$
⇒	length	L	$20\,\mathrm{m}\;(=2\pi R)$	$100\,000\,\mathrm{km}$	$1.5\times10^{8}\,\mathrm{km}$	$4\times10^{4}\,\mathrm{km}$
⇒	particle density	n	$10^{20}\,\mathrm{m}^{-3}$	$10^{16}\,\mathrm{m}^{-3}$	$10^{7}\,\mathrm{m}^{-3}$	$10^{10}\,\mathrm{m}^{-3}$
⇒	magnetic field	B	$3\,\mathrm{T}$ (30 kG)	$0.03\,\mathrm{T}$ (300 G)	$6\times10^{-9}\,\mathrm{T}$	$3\times10^{-5}\,\mathrm{T}$
⇒	temperature	T	$10^{8}\,\mathrm{K}\;(\approx 10\,\mathrm{keV})$	$10^{6}\,\mathrm{K}\;(\approx 100\,\mathrm{eV})$	$10^{5}\,\mathrm{K}$	$10^{4}\,\mathrm{K}$
	density*	$\rho\equiv(A/Z)nm_p$	$1.7\times10^{-7}\,\mathrm{kg\,m^{-3}}$	$1.7\times10^{-11}\,\mathrm{kg\,m^{-3}}$	$1.7\times10^{-20}\,\mathrm{kg\,m^{-3}}$	$1.7\times10^{-17}\,\mathrm{kg\,m^{-3}}$
	pressure	$p\equiv nkT$	$1.4\times10^{5}\,\mathrm{N\,m^{-2}}$	$0.14\,\mathrm{N/\,m^{2}}$	$1.4\times10^{-11}\,\mathrm{N\,m^{-2}}$	$1.4\times10^{-9}\,\mathrm{N\,m^{-2}}$
	‘plasma beta’	$\beta\equiv 2\mu_0 p/B^2$	0.04	0.0004	1	4×10^{-6}
	sound speed*	$v_s\equiv\sqrt{\gamma p/\rho}$	$1.2\times10^{6}\,\mathrm{m\,s^{-1}}$	$120\,\mathrm{km\,s^{-1}}$	$37\,\mathrm{km\,s^{-1}}$	$12\,\mathrm{km\,s^{-1}}$
	Alfvén speed*	$v_A\equiv B/\sqrt{\mu_0\rho}$	$6.5\times10^{6}\,\mathrm{m\,s^{-1}}$	$6500\,\mathrm{km\,s^{-1}}$	$41\,\mathrm{km\,s^{-1}}$	$6500\,\mathrm{km\,s^{-1}}$
	Alfv. transit time	$\tau_A\equiv L/v_A$	$3\,\mu\mathrm{s}$	$15\,\mathrm{s}$	42 days	$6\,\mathrm{s}$
	times of interest	τ	seconds	days	months	hours
	resist. decay time	$\tau_d\equiv\mu_0 a^2/\eta$	16 min	3.2×10^{6} yrs	1.7×10^{9} yrs	1.2×10^{3} yrs
	Spitzer resistivity*)	$\eta_\parallel = 65\,Z\ln\Lambda\,T_e^{-3/2}$	$1.3\times10^{-9}\,\Omega\,\mathrm{m}$	$1.2\times10^{-6}\,\Omega\,\mathrm{m}$	$5.2\times10^{-5}\,\Omega\,\mathrm{m}$	$1.2\times10^{-3}\,\Omega\,\mathrm{m}$
	magn. Reynolds nr.	$R_m\equiv\mu_0 a v_A/\eta_\parallel$	6.3×10^{9}	6.8×10^{13}	1.5×10^{12}	4.1×10^{10}

	radius (km)	*mass* (kg)	*rotation per.* (d)	*distance to Sun* (AU)	*orbital eccentr.*	*orbital per.* (y)
Sun	7.0×10^5	2.0×10^{30}	25–27	–	–	–
Mercury	2.4×10^3	3.3×10^{23}	58.6	0.39	0.206	0.241
Venus	6.1×10^3	4.9×10^{24}	243.0	0.72	0.007	0.615
Earth	6.4×10^3	6.0×10^{24}	1.0	1.0 (def.)	0.017	1.0
Mars	3.4×10^3	6.4×10^{23}	1.03	1.52	0.093	1.88
Jupiter	7.1×10^4	1.9×10^{27}	0.42	5.20	0.048	11.9
Saturn	6.0×10^4	5.7×10^{26}	0.43	9.54	0.056	29.5
Uranus	2.6×10^4	8.7×10^{25}	0.72	19.2	0.047	84.0
Neptune	2.5×10^4	1.0×10^{26}	0.75	30.1	0.009	164.8
Pluto	1.8×10^3	1.0×10^{22}	6.4	39.4	0.249	248.6

$R_\odot = 6.96 \times 10^5\,\mathrm{km}\,,$
$1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km} = 215\,R_\odot\,,$
$1\,\text{light-year} = 9.46 \times 10^{12}\,\mathrm{km} = 6.32 \times 10^4\,\mathrm{AU}\,,$
$1\,\mathrm{pc} = 3.09 \times 10^{13}\,\mathrm{km} = 3.26\,\text{light-years} = 2.06 \times 10^5\,\mathrm{AU}\,.$

	dipole moment (A m^2)	*equatorial magn. field* ($\times 10^{-4}$ T)	*obliquity*	*tilt mag. axis with respect to rot. axis*	*size day-side magnetopause (planet. radii)*
Mercury	4.0×10^{19}	0.003	0.0°	14.0°	1.4 (R_{M})
Venus	$< 1.0 \times 10^{18}$	< 0.0003	177.4° [1]	–	1.1 (R_{V})
Earth	8.1×10^{22}	0.31	23.5°	11.4°	10.4 (R_{E})
Mars	2.3×10^{19}	0.0006	25.2°	–	?
Jupiter	1.5×10^{27}	4.3	3.1°	$-9.6°$ [2]	65 (R_{J})
Saturn	8.6×10^{25}	0.22	26.7°	$-0°$ [2]	20 (R_{S})
Uranus	3.9×10^{24}	0.23	97.9° [1]	$-58.6°$ [2]	18 (R_{U})
Neptune	2.0×10^{24}	0.13	28.8°	$-46.8°$	35 (R_{N})
Pluto	?	?	65°	?	?

[1] retrograde [2] dipole moment in the same direction as rotation vector